EX 1

- $m = 0.100 \cdot 10^{-4}$ $\qquad M = 0.999 \cdot 10^{4}$
- $\text{Spacing}_{[\beta^p, \beta^{p+1}]} = \beta^{p+1-t} = 10^{p-2} = 10 \qquad p = 3 \Rightarrow [10^3, 10^4]$
  $t = 3$
  $\beta = 10$
- $x = 1.01 = 0.101 \cdot 10^1 \in \mathcal{F}$
  $y = 0.1246 \notin \mathcal{F} \qquad fl(y) = 0.125 \cdot 10^0$ approssimazione per eccesso
  $z = 0.1244 \notin \mathcal{F} \qquad fl(z) = 0.124 \cdot 10^0$ approssimazione per difetto

$$\begin{array}{l} 0.125 \cdot 10^0 - \\ 0.124 \cdot 10^0 = \\ \hline 0.001 \cdot 10^0 = 0.1 \cdot 10^{-2} \in \mathcal{F} \end{array} \qquad \begin{array}{l} 0.101 \cdot 10^1 * \\ 0.1 \cdot 10^{-2} = \\ \hline 0.101 \cdot 10^{-2} \in \mathcal{F} \end{array}$$

$$\begin{array}{l} 0.101 \cdot 10^1 * \\ 0.125 \cdot 10^0 = \\ \hline 0.12625 \cdot 10^0 \notin \mathcal{F} \end{array}$$

$\rightsquigarrow fl(0.12625) = 0.126 \cdot 10^0$ approssimazione per difetto

$$\begin{array}{l} 0.126 \cdot 10^0 - \\ 0.125 \cdot 10^0 \\ \hline 0.001 \cdot 10^0 = 0.1 \cdot 10^{-2} \in \mathcal{F} \end{array}$$

$$\begin{array}{l} 0.101 \cdot 10^1 * \\ 0.124 \cdot 10^0 = \\ \hline 0.12524 \cdot 10^0 \notin \mathcal{F} \end{array}$$

$fl(0.12524) = 0.125 \cdot 10^0$ approssimazione per difetto

- Newton $\qquad x_{n+1} = x_n - f_n / f'_n \qquad f(x) = x^2 - 1 \qquad f'(x) = 2x$
  - $x_0 = 1.15 = 0.115 \cdot 10^1 \in \mathcal{F} \qquad x_0^2 = 1.3225 = 0.13225 \cdot 10^1 \notin \mathcal{F} \overset{fl}{\rightsquigarrow} fl\ 0.132 \cdot 10^1$ approssimazione per difetto
    $0.132 \cdot 10^1 - 1 = 0.32 \cdot 10^0 \in \mathcal{F}$
  - $2x_0 = 0.23 \cdot 10^1 \in \mathcal{F}$
  - $0.32 \cdot 10^0 / 0.23 \cdot 10^1 = 0.1391\ldots 10^0 \notin \mathcal{F} \overset{fl}{\rightsquigarrow} 0.139 \cdot 10^0$
  - $0.115 \cdot 10^1 - 0.139 \cdot 10^0 = 0.1011 \cdot 10^1 \notin \mathcal{F} \overset{fl}{\rightsquigarrow} 0.101 \cdot 10^1$

Ex 2

$$A = \begin{bmatrix} a_{11} & & & \\ a_{21} & a_{22} & & 0 \\ \vdots & & \ddots & \\ a_{n1} & \cdots & \cdots & a_{nn} \end{bmatrix}$$

- $$M_1 A = \begin{bmatrix} 1 & & & \\ -\frac{a_{21}}{a_{11}} & 1 & & 0 \\ \vdots & & \ddots & \\ -\frac{a_{n1}}{a_{11}} & & & 1 \end{bmatrix} \begin{bmatrix} a_{11} & & & & \\ a_{21} & a_{22} & & & 0 \\ a_{31} & a_{32} & a_{33} & & \\ \vdots & & & \ddots & \\ a_{n1} & \cdots & \cdots & & a_{nn} \end{bmatrix} = \left[\begin{array}{c|cccc} a_{11} & & 0^T & & \\ \hline 0 & a_{22} & & & \\ \vdots & a_{32} & \ddots & & 0 \\ \vdots & \vdots & & \ddots & \\ 0 & a_{n2} & \cdots & \cdots & a_{nn} \end{array}\right]$$ invariata

$$\Rightarrow \quad A = L_A U_A = \begin{bmatrix} 1 & & & & \\ +\frac{a_{21}}{a_{11}} & 1 & & & \\ \vdots & +\frac{a_{32}}{a_{22}} & 1 & & \\ \vdots & & & \ddots & \\ +\frac{a_{n1}}{a_{11}} & +\frac{a_{n2}}{a_{22}} & \cdots & +\frac{a_{n\,n-1}}{a_{n-1\,n-1}} & 1 \end{bmatrix} \begin{bmatrix} a_{11} & & & \\ & \ddots & & 0 \\ 0 & & \ddots & \\ & & & a_{nn} \end{bmatrix} \qquad U_A = \mathrm{diag}(A)$$

- A^T triangolare superiore $\Rightarrow$ $A^T = L_{A^T} U_{A^T}$ con $L_{A^T} = I$ e $U_{A^T} = A^T$

- $AA^T = L_A U_A L_{A^T} U_{A^T} = L_A \,\mathrm{diag}(A)\, I\, A^T = L_A \cdot \left(\mathrm{diag}(A) A^T\right)$

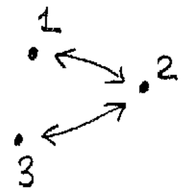
$$\Rightarrow \quad L_{AA^T} = L_A \quad e \quad U_{AA^T} = \mathrm{diag}(A)\, A^T$$

EX 3

Matrice di iterazione di Jacobi

$$P_J = -D^{-1}(L+U) = \begin{bmatrix} 0 & \frac{1}{2+a} & 0 \\ \frac{1}{2+a} & 0 & \frac{1}{2+a} \\ 0 & \frac{1}{2+a} & 0 \end{bmatrix}$$

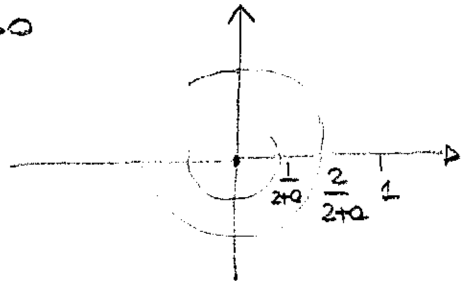
simmetrica irriducibile



$K_{1,3} \quad |z| \leq \frac{1}{2+a}$

$K_2 \quad |z| \leq \frac{2}{2+a}$

Caso $a>0$

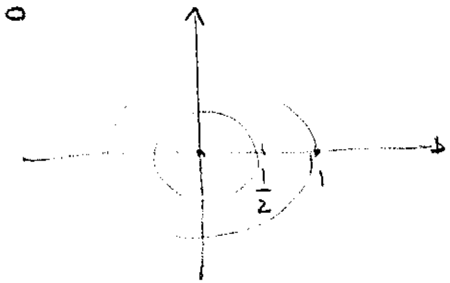


Primo teorema di Gerschgorin

$$\rho(P_J) \leq \frac{2}{2+a} < 1$$ metodo convergente

Oss Si può usare anche il terzo ma la convergenza è già garantita

Caso $a=0$



Primo + terzo teorema di Gerschgorin

$$\rho(P_J) < 1$$

1 e -1 non possono essere autovalori

EX 4

$$\overset{\alpha_2}{y_{n+2}} - \overset{\alpha_1}{(1+a)\, y_{n+1}} + \overset{\alpha_0}{a\, y_n} = h\left(\overset{\beta_1}{f_{n+1}} - \overset{\beta_0}{a f_n}\right) \qquad a \in \mathbb{R} \qquad \text{Esplicito}$$

<u>Consistenza</u>

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h \sum_{j=0}^{k} \beta_j f_{n+j}$$

$\alpha_0 = a$ $\quad$ $\beta_0 = -a$ $\quad$ $k = 2$

$\alpha_1 = -(1+a)$ $\quad$ $\beta_1 = 1$

$\alpha_2 = 1$ $\quad$ $\beta_2 = 0$

$$C_0 = \sum_{j=0}^{k} \alpha_j = a - (1+a) + 1 = 0 \quad \forall a$$

$$C_1 = \sum_{j=0}^{k} j\alpha_j - \beta_j = -(1+a) + 2\cdot 1 - (-a) - 1 = \cancel{-1} \cancel{-a} + \cancel{2} + \cancel{a} \cancel{-1} = 0 \quad \forall a$$

$\Rightarrow$ consistenza di ordine almeno 1 $\forall a$

$$C_2 = \sum_{j=0}^{k} \frac{j^2}{2}\alpha_j - j\beta_j = -\frac{1}{2}(1+a) + \frac{4}{2}\cdot 1 - 1 = -\frac{1}{2} - \frac{a}{2} + 2 - 1 = \frac{1}{2} - \frac{a}{2} = 0 \quad \text{se } a = 1$$

consistenza di ordine almeno 2 per $a = 1$

$$C_3 = \sum_{j=0}^{k} \frac{j^3}{6}\alpha_j - \frac{j^2}{2}\beta_j = -\frac{1}{6}(1+a) + \frac{8}{6}\cdot 1 - \frac{1}{2}\cdot 1 \underset{a=1}{=} -\frac{2}{6} + \frac{8}{6} - \frac{1}{2} = \frac{1}{2} \neq 0$$

per $a = 1$ ordine consistenza $= 2$

<u>0-stabilità</u> Root condition

$$\alpha_2 \zeta^2 + \alpha_1 \zeta + \alpha_0 = \zeta^2 - (1+a)\zeta + a = 0$$

$\zeta_1 = 1$ per consistenza

$\zeta_2 = a \leftarrow \zeta_1\zeta_2 = \frac{a}{1}$ $\Rightarrow$ deve essere $|a| \leq 1$

per $a = 1$ $\zeta_1 = 1$ doppia no 0-stabile $\Rightarrow$ no convergenza

per $a = -1$ $\zeta_2 = -1$, $\zeta_1 = 1$ semplici $\Rightarrow$ 0-stabile $\Rightarrow$ convergente

$\Rightarrow$ Metodo convergente per $-1 \leq a < 1$ di ordine 1

Metodo consistente per $a = 1$ di ordine 2 ma non convergente in quanto non 0-stabile.